*70.Introduction*

Timber production is inherently a risky business. Stumpage price fluctuates all the time, creating uncertainty for forestland owners and managers. Over the years, rotation age determination under uncertainty has remained one of the key topics in forest management and economic. To overcome the stumpage price uncertainty, over the last 30 years, highly sophisticated mathematical tools have been employed to address forest management under price uncertainty. A partial list would include Alvarez and Koskela (2007) and the reference therein, Chladná (2007), Gjolberg and Guttormsen (2002), Hughes (2000), Insley (2002), Insley and Rollins (2005), Limaei and Mohammadi (2021), Lu and Gong (2003), Morck et al. (1989), Rakotoarison and Loisel (2017), Sødal (2002), Thomson (1992) and Willassen (1998). The article by Yousefpour et al (2012) provided a comprehensive review of the relevant literature in the context of dealing with risks and uncertainties caused by climate changes. It is important to note that all of them tried to internalize the effects of price uncertainty.

Brazee and Mendelsohn (1988), hereafter B&M, devised the reservation price method to determine both the value of the land and trees as well as the rotation age. As Plantinga (1998) pointed out, the reservation price of B&M method represents a mechanism for incorporating the option value in determining the optimal rotation length**.**  Despite all previous efforts to address price uncertainty, however, forestland owners have always been functioning as the price taker in the market almost since the beginning of commercial forestry. In other words, forestland owners can only accept the market price. When the market stumpage price is low, forestland owners barely have any choice but wait. In other words, the entire decision process in traditional forestry is passive. In this study, we are introducing a new method that applies real option as a proxy to help forestland owners to actively making forest management decisions. Rather than merely waiting for the good price in the future, one could leverage the financial tools to cover certain downside price risks to manage the price uncertainties more actively than ever. This could bring the entire forest management into a new era.

Conceptually, a put option enables the forestland owners to have the choice but not the obligation of selling a specified amount of timber for a specific price (strike price) by a particular date. An American put option allows the owner of the put option to exercise it any time over its duration while the European put option can only be exercised at the end of the contract. The American put option is chosen in this study because such a purchase eliminates certain level of price uncertainty over that duration of the contract. More importantly, the derived option values enable the forest owners/managers to establish the reservation prices at various stand ages to take advantage of potentially high stumpage prices that might occur.

In this paper, we outsource the downside stumpage price uncertainty with American put options of different lengths to actively establish the option values and the reservation prices. When the spot market price of the stumpage exceeds that of the reservation price, the stand should be harvested to realize superior financial returns. In doing so, timber production becomes an active venture with forestland owners selecting the length of the put option, the starting and ending year of the management window, as well as the strike price. As a result, forestland owners choose a specific set of reservation prices appropriate for their risk preferences and determine the corresponding rotation age and land expectation value accordingly.

To apply this concept to forest management with price uncertainty, the forestland owner could outsource such uncertainty with an American put option O(P1(T1), τ1) Q1(T1) at age τ1, τ1 < T1 with the specified target price of P1(T1) for a specific quantity Q1(T1) by the expiration age T1, T1 ≥t1. In this study, the put option would be purchased at age τ1=15, for an expiration age of T1=70 while the harvest age t1 could be somewhere between 15 and 70. To provide enough time to realize the benefit of the put option, T1 is longer than twice the optimal rotation age under certainty. But since the longer the T1-τ1 is, the more expensive the put option would be. Further, the higher the specified strike price P1(T1) and the larger the specified volume Q1(T1), the costlier the put option would be. Therefore, judicious selection of these variables for the put option represents important management decisions in establishing the reservation prices.

*Simulations and Analyses*

In this section, the results of the reservation prices established with the American put option and that by B&M approach are determined and compared. Once the reservation prices are established at various stand ages, simulations are carried out to determine the average rotation age and LEV1 -- the land expectation value at the beginning of the first rotation under the generalized Faustmann formula (Chang 1998).

(1)

Where

V1(t1) =P1(t1)Q1(t1) represents the stumpage value of a t1-year old stand per acre with P1(t1) being the stumpage price of the t1-year old stand and Q1(t1) being the stand volume per acre of the t1-year old stand.

A1,s represents the miscellaneous annual income or expenses per acre during the first rotation for year s, 0 < s < t1.

C1 represents the regeneration cost per acre of the first rotation.

r1 represents the annual interest rate during the first rotation.

LEV2 represents the land expectation value per acre at the end of the first rotation and the beginning of the second rotation. It embodies all the optimal rotation ages of future rotations and represents the present value of profits from all future rotations.

LEV1 is chosen in this study instead of the classical land expectation value (LEV) proposed by Faustmann (1849) because the stumpage price with all its uncertainty, stand volume, regeneration cost, and interest rates are unlikely to repeat themselves from rotation to rotation forever.

Given the relevant parameters shown in Table 1, the optimal rotation age under certainty will be 31 for loblolly pine and LEV1 of $1921.35/A. An expiration age of 70 is chosen to ensure that it is long enough to capture the benefits of price fluctuations. To calculate the put option value, the average real stumpage price of southern pine in Louisiana from 1956 to 2015 of $169.19/MBF with a standard deviation of $65.73/MBF (Zhang and Chang 2018) is chosen as both the spot and strike prices. As shown in Table 1 with an interest rate of 4%, the option value calculated by the Cox et al. (1979) method, at age 15 the 55-year American put option is valued at $61.6171 per MBF and gradually declines to $0 per MBF at age 70. For any year ω­1, ≤ ω­1 ≤ , the reservation price RP(ω­1) can be calculated as follows:

RP(ω­1) = + (2)

For example, given the settings above, the reservation price at age 15 would be

RP(15)=169.19+ 61.6171\*(83640.72/5236.11) = $1153.46/MBF

This reservation price at age 15 is so high because the option value of $61.6171/MBF must be multiplied by 15.97 (83.641 MBF at age 70 divided by 5.236 MBF at age 15). If the spot price at age 15 is higher than this reservation price, literally with no upside potential and 100% downside risk, an immediate timber harvest would be justified. As the stand age increases, the difference between Q(ω­1) and Q(70) narrows. Over time, the option value decreases, and the reservation price declines. By age 70, with an option value of 0, the reservation price is the same as the target price of $169.19. Compared with the reservation prices obtained by the B&M method, Figure 1 shows that the reservation prices obtained by the 55-year put option approach are higher everywhere than those of B&M. The gap between the two is at its widest initially, then narrows down over time until they converge at age 70.

To examine the impact of the reservation prices of these two methods on the optimal rotation age and LEV1, 50,000 simulations each are carried out. Shown in Table 2 are the average rotation age and LEV1 of the two methods. The 55-year American put option method results in an average optimal rotation age of 46.15 years and an average LEV1 of$2632.72/A. The former is 50% longer than the 31 years under certainty and the latter is 37 % higher than the $1921.35/A under certainty. When compared with the results of the B&M method, however, the former is 10.7 years longer than the 35.45 years of the B&M method and the latter is $249.87/A less than the $2882.59/A of the B&M method. Visual inspection of the reservation price curves of Figure 1 indicates that the reservation price curve of the 55-year American put is tilted backward towards the end of 70 years. Not surprisingly, it results in the much longer average rotation age.

*Rolling put options*

Given that the full-length put option of 55 years creates very high reservation prices during its earlier years and results in longer average rotation age, one may wonder if there is an option that could help forestland owners balance the cost of buying options and getting downside price risk protections. Here we peopose using rolling puts of shorter durations, i.e., instead of buying a full-length put option, one could buy an option with shorter durations to lower the cost. In other words, it would be depending on forestland owners’ preference to decide how long the protection he or she wants given the cost. To this end, ten rolling puts– that of a rolling 10, 13, 15, 17, 19, 20, 21, 23, 25 and 40-year rolling options are explored. For a rolling 10-year put, it will start with a put option from age 15 and ends at age 25, to be followed by a put option from age 16 to age 26 and so on. By age 61, it will then conclude with the last 10 put options of 9, 8, 7, 6, 5, 4, 3, 2, 1 and 0 year. The reservation prices obtained with the rolling 10, and 20-year rolling put options are shown in Table 1 and Figure 1. Due to their shorter option durations, the reservation prices of the 10 and 20-year rolling puts tend to be lower and flatter, which, in turn, affect both the average rotation age and land expectation value.

Table 2 presents the average LEV1 and rotation age plus their standard deviations from 50,000 simulations for rolling put option of various lengths. Among them, the 15-year put rolling option results in the highest average LEV1 of $2872.06/A with an average rotation age to 35.69 years. These results are, essentially, the same as the average LEV1 of $2882.60/A and rotation age of 35.45 years obtained with the B&M method. Interestingly, the 10-year rolling American put option results in a flatter reservation price curve. The average rotation age shortens to a surprising 29.86 years and an average LEV1 of $2771.68/A. The former is even shorter than the rotation age of 31 years while the latter is 44% larger than the LEV1 of $1921.35/A under certainty. Furthermore, the LEV1 with the 10-year rolling put option is only 3.5% less than that of the 15-year rolling put option. As shown in Figure 2, the land expectation value peaks with the 15-rolling put option. Longer rolling put options afterwards result in steadily declining land expectation value, with that of the 40-year rolling put option within $6 of that of the 55-year option.

*Discussion*

Regardless of the duration of the put options, when compared with the outcome under certainty assumption, outsourcing stumpage price uncertainty with a put option enhances the timberland value expressed as LEV1. Moreover, Figure 2 also shows that the duration of the rolling put option and the average LEV1 exhibits a curvilinear relation. Since buying a full length 55-year put option means the forest landowner is extremely risk averse, wanting to cover all downside risks. Buying a shorter length of put option, on the other hand, might provide a choice for people who are less risk averse and choose to cover some risks. In other words, the length of put option provides a way of gauging managers’ risk preference. The curvilinear relation between the land expectation value and the length of the rolling put option indicates that a specific level of risk averse which maximizes LEV1,reaffirming the finding of Zhang and Chang (2018).

Even though the 15-year rolling put option produces the best financial outcome, note that between a rolling option duration of 13 and 20, their land expectation values are within $45/A or 1.6% of that of the 15 years, suggesting that the length of the rolling put option makes a relatively small difference within the neighborhood of the optimal rolling put option length. Further, the surprisingly small difference in LEV1 between a rolling 10-year and 15-year put options of only $101.27/A ($2872.95-2771.68) or 3.5% seems to suggest that for the current example too much risk taking, while not advisable, may not carry a high price. On the other hand, forestland owners may want to shun away from choosing longer rolling puts, which results in significantly lower land expectation values. Surely, additional research is required to confirm if these initial conclusions remain valid for stumpage prices with wider standard deviation. Furthermore, as shown in Table 3, for the 15-year rolling put option, .7% of the time timber harvests occur before age 20; 0.78% of the time occur they occur between 60 and 65 years, and .33% of the time they occur after age 65 for a total 1.81% of the time. The financial impact of changing the starting and ending ages and thus the management window represents a question of high practical relevance. Lastly, field forestry operations often involve annual income such as hunting lease, carbon sequestration, and other payment for ecosystem services as well as annual management expenses. How such inclusions would affect the option value and consequently reservation price, harvest age and the financial outcome of management decisions awaits careful exploration.

*Conclusions*

This article demonstrates that the American put option can be used effectively to outsource stumpage price uncertainty and enhance the land expectation value. The reservation prices derived from the American put option for various stand ages establish benchmarks for forestland owners to determine whether the market price is high enough to justify an immediate timber harvest. With this approach, the forestland owners could realize higher harvest revenue and higher land expectation values to the tune of 40%. This efficacy in enhancing the land expectation value is beyond disputes. The actual figures of the land expectation value reported here are of lesser importance because they depend critically on the interest rate and other input factors used in the calculation.

This method represents a new beginning. With the American put option to determine the option value and reservation price, timberland owners can now actively manage stumpage price uncertainty in timber production. They can also explore different combinations of strike price, interest rate, the length of the management window, and the length of the rolling put option for the desirable reservation prices, resulting in specific land expectation value and rotation length.

Once the owners begin actively selecting the strike price, the length of the management window, and the length of the rolling option, they are no longer price takers. Instead, they become price setters. That would cause a sea change in the stumpage market. As Gong and Löfgren (2007) have shown, its implications on timber supply and social welfare could be profound and need to be fully explored.

Amacher, G. S., Malik, A. S., & Haight, R. G. (2005a). Forest landowner decisions and the value of information

under fire risk. Canadian Journal of Forest Research, 35(11), 2603–2615.

Amacher, G. S., Malik, A. S., & Haight, R. G. (2005b). Not getting burned: The importance of fire prevention in

forest management. Land Economics, 81(2), 284–302

Amacher, G. S., Malik, A. S., & Haight, R. G. (2005a). Forest landowner decisions and the value of information

under fire risk. Canadian Journal of Forest Research, 35(11), 2603–2615.

Amacher, G. S., Malik, A. S., & Haight, R. G. (2005b). Not getting burned: The importance of fire prevention in

forest management. Land Economics, 81(2), 284–302

Amacher, G. S., Malik, A. S., & Haight, R. G. (2005a). Forest landowner decisions and the value of information

under fire risk. Canadian Journal of Forest Research, 35(11), 2603–2615.

Amacher, G. S., Malik, A. S., & Haight, R. G. (2005b). Not getting burned: The importance of fire prevention in

forest management. Land Economics, 81(2)

Literature Cited.

Alvarez, L.H.R. and E. Koskela. 2007. Optimal harvesting under resource stock and price uncertainty. Journal of Economic Dynamics and Control 31: 2461-2485.

Brazee, R. and R. Mendelsohn. 1988. Timber harvesting with fluctuating prices. Forest Science 34(2): 359 – 372.

Chang, S.J. 1998. A generalized Faustmann model for the determination of the optimal harvest age. Canadian Journal of Forest Research 48(5): 652-659.

Chladná Z. 2007. Determination of optimal rotation period under stochastic wood and carbon prices. Forest Policy and Economics 9: 1031-1045.

Cox, J., S. Ross, and M. Rubinstein. 1979. Option pricing: a simplified approach. Journal of Financial Economics 1979: 229-263.

Faustmann, M. 1849. Berechnung des wertes welchen waldboden sowie noch nicht haubare holzbestände für die weldwirtschaft besitzen. Allgemeine Forst-und Jagd-Zeitung 25: 441– 455.

Gjolberg, O., and A. G. Guttormsen. 2002. “Real Options in the Forest: What if Prices are Mean-Reverting?” *Forest Policy and Economics* 4 (1):13-20. https://doi.org/10.1016/s1389-9341(01)00076-4

Gong, P. and K.G. Lofgren. 2007. Market and welfare implication of the reservation price strategy for forest harvest decisions. Journal of Forest Economics 13(4): 217-243.

Hughes, W.R. 2000. Valuing a forest as a call option: the sale of Forest Corporation in New Zealand. Forest science 46(1):32-39.

Insley, M. 2002. A real option approach to the valuation of a forestry investment. Journal of Environmental Economics and Management 44: 471 – 492.

Insley, M. and K. Rollins. 2005. On solving multirotational timber harvesting problem with stochastic prices: a linear complementarity formulation. American Journal of Agricultural economics 87(3): 735-755.

Limaei, S. M., and Z. Mohammadi. 2021. “Optimal Forest Management Using Stochastic Dynamic Programming Approach - A Case Study from the Hyrcanian Forests of Iran.” *Journal of Sustainable Forestry*. https://doi.org/10.1080/10549811.2021.1961277

Lu, F., and P. Gong. 2003. Optimal stocking level and final harvest age with stochastic prices. Journal of Forest Economics, 9(2): 119–136.

Morck, R., E. Schwartz, and D. Strangeland. 1989. The valuation of forest resources under stochastic prices and inventories. Journal of financial and quantitative analysis 24(4): 473-487.

Plantinga, A., 1998. The optimal timber rotation: an option value approach. For Sci 44(2):192–202

Rakotoarison, H., and P. Loisel. 2017. The Faustmann model under storm risk and price uncertainty: A case study of European beech in Northwestern France. Forest Policy and Economics 81: 30-37.

Sødal, S. 2002. The stochastic rotational problem: a comment. Journal of Economic Dynamics and Control. 26: 509-515.

Thompson, T. 1992. Optimal forest rotation when stumpage prices follow a diffusion process. Land Economics 68(3): 329-342.

Willassen, Y. 1998. The stochastic rotation problem: A generalization of Faustmann’s formula to stochastic forest growth. Journal of Economic Dynamics and Control 22: 573-596.

Yousefpour, R., J.B. Jacobsen, B.J. Thorsen, H. Meilby, M. Hanewinkel, and K. Oehler. 2012. A review of decision-making approaches to handle uncertainty and risk in adaptive forest management under climate change. Annals of Forest Science 69: 1- 15.

Zhang, F., and S.J. Chang. 2018. Measuring the impact of risk preference on land valuation: Evidence from forest management. L:and Economics 94(3): 425-436.